National Mathematics Curriculum: Framing paper

For consultation: November 2008 – 28 February 2009

CONTENTS

PREAMBLE

1 The *National Mathematics Curriculum: Framing paper* proposes broad directions for what teachers should teach and young people should learn in the national mathematics curriculum from Kindergarten[1] to Year 12.

2 The purpose of this paper is to generate broad-ranging discussions about curriculum development. The paper is posted on the National Curriculum Board's website (www.ncb.org.au) with an invitation to all those interested to provide feedback and advice up to 28 February 2009.

3 The Board will then examine all feedback and determine its final recommendations to guide curriculum development.

Process to develop the *National Mathematics Curriculum: Framing paper*

4 The National Curriculum Board began its consultation with the publication of the *National Curriculum Development Paper* on its website. This paper described the context of its work and set down questions that needed to be answered to determine the kind of curriculum that would be developed.

5 *The Shape of the National Curriculum: A Proposal for Discussion* was developed following feedback from the 'Into the Future: National Curriculum Board Forum' and state and territory consultation forums. Appendix 1 provides details about its principles and specifications for curriculum development. This paper is posted on the Board's website with an invitation to anyone interested to provide feedback during Term 4, 2008.

6 The Board began work on framing the national mathematics curriculum by recruiting a writer who worked with a small advisory group (see Appendix 2) to draft an initial advice paper that provided a broad scope and sequence from Kindergarten to Year 12.

7 This initial advice paper was discussed at a national forum in October. On the day after the forum a small group of nominees from the Australian Association of Mathematics Teachers met with the writer to discuss the feedback from the forum and its implications for developing the curriculum.

8 The *National Mathematics Curriculum: Framing paper* is built on this initial advice, advisory group feedback, submissions through the Board's website, individual responses by academics and teachers, responses from national, state and territory forums and responses received through email and letters.

9 The *National Mathematics Curriculum: Framing paper* is best read in conjunction with *The Shape of the National Curriculum: A Proposal for Discussion* to provide a context for the shape of the curriculum overall. The paper focuses on what the content for the national mathematics curriculum might be; assessment and pedagogy, although key considerations for any curriculum, are addressed only briefly in the paper.

[1] 'Kindergarten refers to the first year of school. In some jurisdictions this is called 'Reception' or 'Preparatory'.

Providing feedback about the *National Mathematics Curriculum: Framing paper*

10 The Board welcomes feedback on this paper. Survey questions are included in Appendix 3 and there are several ways to participate. Survey forms can be emailed to *feedback@ncb.org.au* and written feedback can be mailed to: National Curriculum Board Feedback, PO Box 177, Carlton South, Victoria 3053.

11 From 21 November online feedback for this paper can be submitted through the Board's website link:

http://www.ncb.org.au/get_involved/subscribe/ways_of_participating.html

Register from this link. Once you have joined, a username and password provide easy access to online surveys, discussions and summaries of feedback comments. This is an opportunity to be fully involved and up-to-date with national curriculum development.

INTRODUCTION

12 The national mathematics curriculum will be the basis of planning, teaching, and assessment of school mathematics, and be useful for, and useable by, experienced and less experienced teachers of K–12 mathematics.

13 There have been two important reports this year relevant to the development of this paper: the Australian *National Numeracy Review Report* (NNR 2008); and *Foundations for Success: The final report of the National Mathematics Advisory Panel* (NMAP 2008) from the United States. Although both had some emphasis on what research says about learning, teaching and teacher education, they each contribute to current considerations of the mathematics curriculum, particularly in describing the goals and intended emphases.

14 The obvious imperative to create a futures-oriented curriculum provides a major opportunity to lead improved teaching and learning. This futures orientation includes the consideration that society will be complex, with workers competing in a global market, needing to know how to learn, adapt, create, communicate, and interpret and use information critically.

15 In summary, this framing paper argues that:

- mathematics is important for all citizens
- some students are currently excluded from effective mathematics study, and the curriculum and school structure should seek to overcome this
- a futures orientation should be evident in both the emphasis on thinking and creativity, and in the embedding of appropriate use of digital technological tools
- numeracy should be both embedded and specifically identified within the mathematics curriculum
- all aspects of the curriculum should be clearly and succinctly described
- more important topics should be emphasised, with a goal of reducing the extent to which teachers feel the need to rush from topic to topic
- advanced students can be extended appropriately using challenging problems within current topics.

16 Specific aspects within the three content strands to be outlined later will be described for each stage of schooling. For each of the four proficiency strands proposed, expectations will be delineated, along with defining levels of numeracy.

AIMS

17 Mathematics holds a central place in school curriculums not only because it is fundamental to the education of students, but also because it is central to the development of our society and our global competitiveness.

18 Building on the draft *National Declaration on Educational Goals for Young Australians*, a fundamental goal of the mathematics curriculum is to educate students to be active, thinking citizens, interpreting the world mathematically, and using mathematics to help form their predictions and decisions about personal and financial priorities. In a democratic

society, many substantial community, social and scientific issues are raised or influenced by public opinion, so it is important that citizens can critically examine those issues from mathematical perspectives. In addition, mathematics has its own value and beauty and it is intended that students will appreciate the elegance and power of mathematical thinking, experience mathematics as enjoyable, and encounter teachers who communicate this enjoyment.

19 If Australia's future citizens are to be sufficiently well educated mathematically for the development of society and to ensure international competitiveness, two needs must be met. The first is the need not only for adequate numbers of mathematics specialists operating at best international levels, capable of generating the next level of knowledge and invention, but also for mathematically expert professionals such as engineers, economists, scientists, social scientists, and planners. The second need is for the workforce to be appropriately educated in mathematics to contribute productively in an ever-changing global economy, with rapid revolutions in technology and both global and local social challenges. An economy competing globally requires substantial numbers of proficient workers able to learn, adapt, create, interpret and analyse mathematical information.

20 This critical importance of mathematics will be assumed by the mathematics curriculum but it also needs to be reflected in the time and emphasis allocated to mathematics learning in schools and to the study of mathematics teaching in teacher education programs, in resources allocated to the support of the implementation of the curriculum, and in the promotion of the value of the study of mathematics.

TERMS USED IN THIS PAPER

21 This paper uses three terms that together describe the mathematics curriculum: *content strands*, *proficiency strands* and *numeracy*. The cornerstone of mathematics is its interconnectedness, and while these distinctions are somewhat artificial, they facilitate the organisation of the curriculum in a form that will enable the achievement of the aims described in this paper.

Content strands

22 Mathematics curriculums around the world commonly consist of coherent sets of topics. Although there are some variations, the most common categorisations of the basic content strands across the Australian jurisdictions in the compulsory years are: Number, Measurement, Space, Chance and data, and Algebra. For mathematical and pedagogical reasons, it is proposed that the national mathematics curriculum includes three content strands: Number and algebra, Measurement and geometry, and Statistics and probability.

> *Number and algebra*: In this content strand the concentration in the early years will be on number, and near the end of the compulsory years there will be emphasis on algebra. Recent research has emphasised the connections between these aspects. An algebraic perspective can enrich the teaching of number in the middle and later primary years, and the integration of number and algebra, especially representations of relationships, can give more meaning to the study of algebra in the secondary years. This combination incorporates aspects described as pattern and/or structure and includes functions, sets and logic.

Measurement and geometry: While there are some aspects of geometry that have limited connection to measurement, and vice versa, there are also topics in both for which there is substantial overlap, including newer topics such as networks. In many curriculums the term *space* is used to cover mathematical concepts of shape and location. Yet many aspects of location, for example maps, scales and bearings, are aligned with measurement, and the term *geometry* is more descriptive for the study of properties of shapes, and also gives prominence to logical definitions and justification.

Statistics and probability: Although teachers are familiar with the terms *data* and *chance*, *statistics* and *probability* more adequately describe the nature of the learning goals and types of student activity. For example, it is not enough to construct or summarise data, it is important to represent, interpret and analyse it. Likewise, probability communicates that this study is more than the chance that something will happen. The terms provide for the continuity of content to the end of the secondary years and acknowledge the increasing importance and emphasis of these aspects at all levels of study.

23 The names of these content strands refer to substantial mathematics sub-disciplines and so reflect more accurately the purpose of their study. The reduction to three content strands will allow greater coherence within strands, will facilitate the building of connections between related topics within and across strands, and will support a clear and succinct description of the curriculum as discussed below.

Proficiency strands

24 In many jurisdictions the term *working mathematically* is used to describe applications or actions of mathematics. This term does not encompass the full range of desired actions nor does it allow for the specification of the standards and expectations for those actions. It is proposed that the national mathematics curriculum use the four proficiency strands of Understanding, Fluency, Problem solving, and Reasoning, adapted from the recommendations in *Adding it Up* (Kilpatrick, Swafford & Findell 2001), to elaborate expectations for these actions:

Understanding (conceptual understanding) includes building robust knowledge of adaptable and transferable mathematical concepts, the making of connections between related concepts, the confidence to use the familiar to develop new ideas, and the 'why' as well as the 'how' of mathematics.

Fluency (procedural fluency) includes skill in choosing appropriate procedures, carrying out procedures flexibly, accurately, efficiently and appropriately, and in addition, recalling factual knowledge and concepts readily.

Problem solving (strategic competence) includes the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively.

Reasoning (adaptive reasoning) includes the capacity for logical thought and actions such as analysing, proving, evaluating, explaining, inferring, justifying, and generalising.

25 Expectations for these four proficiency strands will be elaborated to inform teaching and assessment. There are specific topics for which understanding is critical (e.g. decimal place value, 2D–3D relationships) and others for which standards for fluency will be specified (e.g. mental calculation, using Pythagoras' theorem). Expectations for proficiency in problem solving (e.g. representing situations diagrammatically) and reasoning (e.g. justifying solutions) will also be specified noting that these are central to ensuring a futures orientation to the curriculum.

Numeracy

26 To provide teachers and students with guidance on numeracy, and to resolve the tension between recognising the importance of numeracy perspectives and avoiding an artificial distinction between it and mathematics, authentic proficiency standards for numeracy will be specified as part of the mathematics curriculum.

27 To elaborate the meaning of numeracy it is helpful to draw on carefully determined definitions. One useful definition of numeracy cited in the NNR (2008) is:

> Numeracy is the effective use of mathematics to meet the general demands of life at school and at home, in paid work, and for participation in community and civic life. (MCEETYA Benchmarking Task Force 1997:4)

28 Numeracy includes capacities that enhance the lives of individuals by enabling them to interact with the world in quantitative terms, communicate mathematically, and analyse and interpret everyday information that is represented mathematically. It incorporates aspects such as number sense, measurement, estimating quantities, bearings, map reading, networks, properties of shapes, and personal finance and budgeting. Numeracy also includes the mathematics used by professionals such as economists, psychologists, architects and engineers, the mathematics that is useful in learning disciplines such as geography, chemistry, physics and electronics, and the everyday vocational mathematics used in fields such as building, sports, health and catering. It involves aspects of accurate measurement, ratio, rates, percentages, using and manipulating formulas, the mathematics of finance, modelling and representing relationships especially graphically, and representing and interpreting sophisticated data.

CONSIDERATIONS

29 In the development of the national mathematics curriculum, some key considerations are equity and opportunity (including the place of numeracy), connections to other learning areas, clarity and succinctness, thinning-out the 'crowded curriculum', and recognising the role of digital technologies.

Equity and opportunity

30 An unintended effect of current classroom practice has been to exclude some students from future mathematics study. The goal of equity of opportunity is central to the construction of the mathematics curriculum. This includes consideration of the need to engage more students, the way particular groups have been excluded, and the challenge posed by creating opportunity.

The need to engage more students

31 The personal and community advantages of successful mathematics learning can only be realised through successful participation and engagement. Although there are challenges at all stages of schooling, participation is most at threat in the middle years. Student disengagement at these levels can be attributed to irrelevant curriculums, missed opportunities in earlier years, ineffectual learning and teaching processes, and perhaps the students' stages of physical development. Alienation of some students from mathematics has been a concern for many years, but changing cultural and technological conditions make it imperative that we reverse this trend. The key issue, for this discussion, is that such students' experience of mathematics is alienating and limited. For example, the *Third International Mathematics and Science Study* (TIMSS) Video Study (Hollingsworth, Lokan, & McCrae 2003) reported that in the Australian Year 8 lessons:

- more than three-quarters of the problems used by teachers were low in complexity (requiring four or fewer steps to solve)
- most problems involved emphasis on procedural fluency
- only one-quarter of problems used any real-life connections.

32 This disengagement in the middle years has flow-on effects. One effect is that, while the overall proportion of students studying mathematics at Year 12 is steady, the decline in participation of students in specialised mathematics studies in most jurisdictions is a concern (Barrington 2006). The NNR (2008) reported two linked issues: the first is a decline in students undertaking major sequences in tertiary mathematics study; and the second is the shortage of qualified secondary mathematics teachers, and the resultant numbers of non-specialist mathematics teachers in junior secondary years. Operating together these factors are cyclical: high-quality teachers can support students in meaningful and productive mathematics learning in the key middle years, and so more students will retain aspirations for further mathematics study, and so it goes on. The curriculum must be designed to overcome this problem.

Ensuring inclusion of all groups

33 A fundamental educational principle is that schooling should create opportunities for every student. There are two aspects to this. One is the need to ensure that options for every student are preserved as long as possible given the obvious critical importance of mathematics achievement in providing access to further study and employment and in developing numerate citizens. The second aspect is the differential achievement among particular groups of students. For example, the following figures are extracted from the report on the PISA 2006 results (Thompson & De Bortoli 2007) relating to numeracy (their term was *mathematical literac*y) of Australian 15-year-olds, comparing the responses of the commonly discussed equity groups. Table 1 compares the achievement of students based on their socioeconomic background.

Table 1: Percentage of Australian students from particular socioeconomic backgrounds in highest and lowest levels of PISA numeracy achievement

	Percent at the highest level	Percent at level 1 or below
Low SES quartile	6	22
High SES quartile	29	5

Similar differences are evident when comparing non-Indigenous and Indigenous achievement, and there are also differences in achievement levels between metropolitan, regional and remote students and, to a lesser extent, between boys and girls.

34 Numeracy (and other academic) achievement seems very much related to SES background (and cultural and geographic factors in other data), which is contrary to a fundamental ethos of Australian education, that of creating opportunities for all students.

35 The differences between the achievements of students at opposite ends of the SES scale are substantial. Those achieving only PISA level 1 are responding at a very low level for 15-year-olds, would have great difficulty coping with the demands of school without specific support, and would have a restricted set of work choices available to them once they leave school. Yet those achieving at the highest level are progressing at the best international standards. It is tempting to cater for the spread of achievement by differentiating opportunities, but it is essential that no barriers to progression in mathematics be imposed before the senior secondary years.

The challenge of creating opportunity

36 The development of the national mathematics curriculum offers a wonderful opportunity to revitalise the experience of all mathematics learners in a way that respects equity considerations. A key first step is to affirm a commitment to ensuring that all students experience the full mathematics curriculum until the end of Year 9, with mathematics being compulsory in Year 10, and with schools developing relevant options preserving for all students the possibility of mathematics study in Year 11. This signals to systems and schools the requirement to ensure structures are inclusive and that support is available for students who need it, and also signals to students that opting out should not be one of their choices.

37 One aspect of making the mathematics curriculum accessible is to emphasise the relevance of the content to students. The specification of proficiency expectations for numeracy will assist in this: for example, any aspects of mathematics for middle years students can be introduced by drawing on practical situations and so the purpose of the study is more obvious, and the mathematics is made more meaningful.

38 The curriculum must also provide access to future mathematics study. It is essential, for example, that all students have the opportunity to study algebra and geometry. The NMAP (2008) argues that participation in algebra, for example, is connected to finishing high school; failing to graduate from high school is associated with under-participation in the workforce and high dependence on welfare. The study of algebra clearly lays the foundations not only for specialised mathematics study but also for vocational aspects of numeracy. Yet the study of algebra represents a challenge for many students during the compulsory years, and serves to exclude some students from further options. In other words, successful study of algebra is a gateway to opportunity, and unsuccessful study of algebra can be the start of alienation from mathematics.

39 There is now an opportunity to rethink the curriculum in the early secondary years. The intention is to increase student access to relevant and important mathematics, with a particular focus on ensuring that algebra and geometry are developed in meaningful and interesting ways. There are specific implications in this for the upper primary curriculum.

Connections to other learning areas

40 Not all learning can be captured within areas into which the school curriculum has traditionally been divided. The national mathematics curriculum will explicitly accommodate the relevant general capabilities.

41 The NNR (2008) identified numeracy as requiring an across-the-school commitment, including mathematical, strategic and contextual aspects. This across-the-school commitment will be managed by including specific reference to other curriculum areas in the mathematics curriculum, and identification of key numeracy capacities in the descriptions of other curriculum areas being developed. For example, the following are indications of some of the numeracy perspectives relevant to history, English, and science.

History: Learning in the subject of history includes interpreting large numbers such as are associated with population statistics and growth, financial data, figures for exports and imports, immigration statistics, mortality rates, war enlistments and casualty figures, chance events, correlation and causation, interpreting and representing a range of forms of data, imagining timelines and timeframes to reconcile relativities of related events, and the perception and spatial visualisation required for geopolitical considerations, such as changes in borders of states and in ecology.

English: One aspect of the link with English and literacy is that, along with other elements of study, numeracy can be understood and acquired only within the context of the social, cultural, political, economic and historical practices to which it is integral. Students need to be able to draw on quantitative and spatial information to derive meaning from certain types of texts encountered in the subject of English.

Science: Practical work and problem solving across all the sciences require the capacity to: organise and represent data in a range of forms; plot, interpret and extrapolate graphs; estimate and solve ratio problems; use formulas flexibly in a range of situations; perform unit conversions; and use and interpret rates including concentrations, sampling, scientific notation, and significant figures.

42 It is proposed that such perspectives be evident in both the mathematics curriculum document and in the document of the other relevant disciplines. All four curriculum documents should ensure alignment of cross-curriculum aspects of numeracy (and literacy).

Clarity and succinctness

43 The form of presentation of the curriculum will be critical to its successful implementation. The experience of many users of curriculum documents in the various jurisdictions is that the documents are long, complex, written in convoluted language, with ambiguous category descriptors in which it is difficult to identify key ideas.

44 The current diversity in terminology adds to complexity and means that many of the high-quality resources produced in various jurisdictions are difficult to transfer to other

contexts. National curriculum documents will be written so that they are accessible to teachers and will help define a language for communication about the curriculum.

45 Documents should communicate succinctly the important ideas of the curriculum. Hattie and Timperley (2007) reviewed a wide range of studies and found that teacher feedback to students is a key determinant of effective learning and that feedback involves making explicit to students what they should be doing, how they are performing, and what is the next phase in their learning. Teachers do this while they are interacting with students, and so need to know the purpose of the current student activity, the expected standards for performance, and subsequent learning goals. A clearly, succinctly written curriculum will assist in this. Excessive elaboration is not necessarily in the best interests of students, can result in compartmentalisation of content and may result in less clarity in the focus of teaching.

Thinning-out the 'crowded curriculum'

46 Many mathematics teachers report that the scope of the curriculum creates pressures to move on to new topics before students have mastered the current one. The NMAP (2008) argued that 'the mathematics curriculum in Grades Pre-K to 8 should be streamlined and should emphasise a well-defined set of the most critical topics in the early grades' (p. 11). When all aspects are presented as though of equal importance, this does not help teachers to appreciate short- and long-term goals, and to identify the key ideas. It is possible to reduce some of the crowding by dealing with complementary concepts together, but there may still be a need for the identification of core topics, or other mechanisms that can allow teachers to feel less hurried.

47 It may be possible to present the curriculum in a way that reduces this sense of being rushed. The NMAP (2008) argued that 'a focused, coherent progression of mathematics learning, with an emphasis on proficiency with key topics should become the norm of elementary and middle school mathematics curriculums' (p. xvi). Not all students progress through aspects of mathematics in the same sequence, and the pathway from one key idea to another is not the same for all students. Further, students tend not to have consistent performance across all aspects of mathematics. The goal is to specify the major learning goals for each content strand, noting that this is a complex task and is not a feature of most current curriculums.

48 The rate of development of the curriculum should enable teachers to extend students in more depth in key topics, and one of the challenges will be to identify which are those more important topics. Fractions and decimals are examples of those more important topics, as are the principles of measurement. Long division is an example of a topic which could be given less emphasis. As an example of how advanced students might be extended in a basic topic, such students could be posed a question like: 'Can you describe some shapes that have the same number of perimeter units as area units?' This creates opportunities for examination of a range of shapes, for use of algebraic methods, and even the historical dimension of this problem.

49 For the purposes of curriculum documentation, learning for most topics can be considered to occur along a continuum, although not necessarily with development at a regular rate. It is essential that this continuum be developed from early to later stages, rather than the reverse, as seems to be the *de facto* situation currently in the secondary years.

The role of digital technologies

50 An important consideration in the structuring of the curriculum is to embed digital technologies so that they are not optional extras. Digital technologies allow new approaches to explaining and presenting mathematics, as well as assisting in connecting representations and thus deepening understanding. The continuing evolution of digital technologies has progressively changed the work of mathematicians and school mathematics (consider the use of logarithm tables and the slide rule), and the curriculum must continue to adapt. Digital technologies are now more powerful, accessible and pervasive. For example, modern mathematical technologies — hand-held devices or computer software — support numerical, statistical, graphical, symbolic, geometric and text functionalities. These may be used separately or in combination. Thus, a student could readily explore various aspects of the behaviour of a function or relation numerically, graphically, geometrically and algebraically using such technologies. These approaches allow greater attention to meaning, transfer, connections and applications. Digital technologies can make previously inaccessible mathematics accessible, and enhance the potential for teachers to make mathematics interesting to more students, including the use of realistic data and examples. The embedding of digital technologies is applicable at all levels of the curriculum and is particularly relevant given the obvious capacity of school students to use digital, information and communication technologies.

51 At the same time, the curriculum will advise parents and teachers on standards and expectations for aspects of mathematics which are better done mentally, and the need for students to make appropriate choices about when to use technology. To give just one instance, it is reasonable to expect that school leavers will choose mental calculation to multiply or divide by 10 or 100, or calculate a 10 percent tip, and that nearly all will be able to accurately estimate 15 percent of a quantity.

STRUCTURE OF THE CURRICULUM

52 This section outlines the implications for the structure of the curriculum and for the emphases across four stages of schooling.

53 The curriculum in the compulsory years should be equitable and create opportunities for learning, with a futures orientation incorporating appropriate digital technological tools. It should be clearly and succinctly described, with key elements identified, using three content strands, and explicit expectations for four proficiency strands along with the key developmental aspects of numeracy.

54 For each cluster of years (K–2, 3–4 etc.), the individual elements or topics of the three content strands — Number and algebra, Measurement and geometry, and Statistics and probability — should be presented, with the key ideas being given prominence, along with opportunity for their elaboration. The initial identification of the key ideas should be written so that it is adequate for experienced teachers, and there should be some additional descriptions suitable for less experienced mathematics teachers. It is noted that primary teachers are responsible for teaching across a range of learning areas, and so clarity and succinctness will be helpful for them.

55 Consider, for example, a possible description of the key aspects of learning subtraction. Assuming that children have experienced preliminary ideas including counting forwards, ordering numbers, seeing that there is the same number of objects however they are arranged (conservation), and immediate recognition of small collections of objects (subitising), the key aspects of the next phase in learning subtraction (relevant to K–2) can be described by using these four aspects:

- backwards counting
- modelling situations in which one part of the whole is unknown
- number strategies that are useful for subtraction
- solving subtraction word problems.

56 It is argued that these aspects are a substantially simplified way of describing the elements of learning subtraction than is used in most curriculums, and that these aspects are comprehensive as well as succinct, and give teachers clear indication of the nature of student experiences in these years. It is noted that together the four aspects represent preparation for more formal subtraction experiences that come subsequently. It is not intended that these represent a strict sequence, but that the four aspects together constitute this phase of learning subtraction. The information under these four aspects may be all the information experienced teachers need, and further brief elaboration of each can be provided for less experienced mathematics teachers. An example of elaboration of the aspect 'number strategies that are useful for subtraction' could be:

> Key facts that form the basis of strategies for subtraction are: taking away 1 and taking away 2; what do we need to build to 10 (6 + ? = 10, etc.); doubles (the idea that if 3 + 3 is known then 6 – 3 is also known); and subtracting 10 and 9.

57 Expectations in each of the proficiency strands — Understanding, Fluency, Problem solving, and Reasoning — will be specified for each content strand for each level. In addition, expectations for proficiency in numeracy will be defined. These will establish the respective achievement standards for the curriculum, and will be used as the basis of national assessments and reporting. It is noted that the Fluency strand has well-established

expectations, so the challenge for the writers will be to define expectations in the other proficiency strands and in numeracy. The following is an example of how proficiency for Understanding might be specified:

> Within the range of numbers from 0 to 99, students see connections between take away, less than, difference between.
>
> Students connect subtraction with other forms of representations, such as empty number lines.
>
> Student see connections between addition and subtraction. They understand that if they know 7 + 8 = 15, they also know:
>
> 15 – 7 = ?   15 – 8 = ?
>
> 15 – ? = 7   15 – ? = 8
>
> ? – 7 = 8   ? – 8 = 7

58 It is noted that the expectations will include indication of proficiency in aspects that use technology, and expectations for proficiency without using technology.

59 Taken together, this means that the curriculum will include the three content strands and elaborations, along with expectations for proficiency in the four proficiency strands and numeracy. If published in a digital form, the first level of detail of the content strands can be specified, with the elaboration accessed by links, and likewise the expectations for the four proficiency strands and numeracy also accessed by links.

60 As an aside, the effective use of ICT as the medium of publication may help in developing a flexible, adaptable, and scalable curriculum, with systems, schools and teachers being able to choose their own form and format of presentation. A flexible form of publication will allow linking of relevant supporting resources, and encourage the development of those resources.

Stages of schooling

61 Although it is proposed that the curriculum will be represented year by year, this document provides an outline across four stages of schooling:

Stage 1, which typically involves students from 5 to 8 years of age

Stage 2, which typically involves students from 8 to 12 years of age

Stage 3, which typically involves students from 12 to 15 years of age

Stage 4, which typically involves students from 15 to 18 years of age.

62 For each stage of schooling the paper discusses how considerations dealt with in the previous section and the structure of the curriculum might apply for each stage of schooling.

Stage 1 (typically from 5 to 8 years of age)

Curriculum focus

63 These years lay the foundation for learning mathematics. The essential ingredient is focused and active experiences with the key ideas. It is assumed that the curriculum at this stage will build on the national Early Years Learning Framework currently in development. Many ideas are best introduced using real or physical models at this level, with emphasis on the development of the relevant language. The Australian Association of Mathematics Teachers (AAMT) and Early Childhood Australia (2006, p. 1) argued that children at this level can access ‘powerful mathematical ideas’ that are ‘relevant to their current lives’, and that it is the relevance to them of this learning that prepares them for the following years. The challenges for the writers include identifying the key ideas, describing them succinctly, and indicating the respective emphases for the three content strands. As with the other stages, these three strands are not of equal importance and teachers need guidelines for the appropriate emphases.

Stage 2 (typically from 8 to 12 years of age)

Curriculum focus

64 The twin challenges of making the curriculum engaging and preparing students for future study become evident at Stage 2. The AAMT (2005) vision for quality mathematics in the middle years notes the importance of students studying coherent, meaningful and purposeful mathematics that is relevant to their lives. Students still require active experiences that allow them to construct key mathematical ideas, but there is a trend to move to using models, pictures and symbols to represent these ideas. The curriculum will establish the key developmental ideas, to extend the number learning and measurement from Stage 1, and to lay the foundations for future studies, particularly algebra. The introduction of fractions and decimals is important and represents a key challenge at this stage. There are also opportunities in emphasising patterns and describing relationships.

Stage 3 (typically from 12 to 15 years of age)

Curriculum focus

65 This paper proposes that the mathematics curriculum will be inclusive of all students to the end of Year 10 and that mathematics should be compulsory. It is expected that schools will offer students relevant options in Year 10 while preserving the possibility for all students to choose an appropriate mathematics study in Year 11.

66 One goal is to engage learners by ensuring the content is meaningful and relevant to their lives. Another goal is to lay the foundations for future studies, including introducing all students to the power of algebra and to aspects of geometry.

67 The intention is that the curriculum will list fewer detailed topics and encourage the development of important ideas in more depth. An obvious concern is the preparation of

students who are intending to continue studying mathematics in the post-compulsory years. It is argued that it is possible to extend the more mathematically able students appropriately using challenges and extensions within the current topics (as distinct from moving on to more advanced topics quickly), and the expectations for proficiency can reflect this.

Stage 4 (typically from 15 to 18 years of age)

Curriculum focus

68 The specification of the curriculum in the senior secondary years has conventionally taken a different form from the compulsory years of schooling. There are important reasons for this, including the elective and specialised nature of such studies. It has been common to determine senior secondary curriculum content using a ‘top down’ approach. Given that the implementation of the national senior secondary curriculum will take more time, it is intended that the specifics of the senior secondary curriculums be specified after the content for the compulsory years has been determined, and build on these specifications.

69 Despite the emphasis by some commentators on differences between the jurisdictions, there is substantial commonality in approaches to senior secondary mathematics study, as identified by Barrington (2006) who reported on enrolments in senior secondary mathematics courses and summarised the choices of students at three levels.

70 One level was for those students who study what Barrington termed *elementary*. The largest enrolments of such students are in General Mathematics (NSW), Further Mathematics (Vic), and Mathematics A (Qld), each of which count toward tertiary selection. The use of the term *elementary* is misleading, and such courses should be designed to be substantial as is currently the case in most jurisdictions. Given that more than half of all Year 12 students choose these courses, this level seems an appropriate inclusion in the national mathematics curriculum. Based on current arrangements the content might include topics such as business or financial mathematics, probability, statistics and measurement, and in some places includes topics like navigation, matrixes, networks, and applied geometry.

71 The next level of courses in the Barrington classification is *intermediate*. The most common descriptor is Mathematical Methods, and other terms are Mathematics (NSW), Mathematics B (Qld), Applicable Mathematics (WA), and Mathematics Studies (NT). These courses provide a substantial development of mathematical knowledge suitable for many students, including those intending to study mathematics at university, and common topics include graphs and relationships, calculus, and statistics focusing on distributions. Some such courses allow appropriate use of computer algebra system calculators which will be an expectation in the new national mathematics curriculum.

72 The third level is described as advanced, with the most common descriptor being Specialist Mathematics, and other terms being Mathematics Extension (1 and 2) (NSW), Mathematics C (Qld), and Calculus (WA). These courses are intended for students with a strong interest in mathematics including those expecting to study mathematics and engineering at university. They commonly include topics like complex numbers, vectors with related trigonometry and kinematics, mechanics, and build on the calculus from the intermediate subject.

73 There are many other offerings at the senior secondary level, designed for students pursuing vocational pathways but not used for university selection, that can also be included.

74 Given the commonality in approach across the jurisdictions, the three Barrington categories, along with a non-tertiary entrance vocational option, provide a useful starting point for development of these national post-compulsory courses.

PEDAGOGY AND ASSESSMENT

75 The preceding discussion on the content and organisation of a national mathematics curriculum is based on some pedagogical assumptions, which include that:

- it is preferable for students to study fewer aspects in more depth rather than studying more aspects superficially
- challenging problems can be posed using basic content, and content acceleration may not be the best way to extend the best students
- effective sets of ideas with goals for key phases specified are preferable to disconnected experiences, even though they may be rich ones
- teachers can make informed classroom decisions interactively if they are aware of the development of key ideas, and a clear succinct description will assist in this
- effective use of digital technologies can enhance the relevance of the content and processes for learning
- teachers can make mathematics inclusive by using engaging experiences that can be differentiated both for students experiencing difficulty and those who can complete the tasks easily.

76 It is assumed that teachers will use a variety of mathematical task types including those that give students choice of approach and those for which there is an optimal strategy; those for which there are various possible solutions and those which have a single correct answer; those that prompt the development and use of mathematical models; those that incorporate ideas across content strands; and those that require thinking in more than one discipline.

77 Another underlying assumption is that specifying expectations in the four proficiency strands and numeracy can help in focussing their teaching. Teachers should base their teaching on what the students already know, should make explicit the subsequent key ideas, should ensure tasks are posed at an appropriate level of challenge, and should offer feedback on activities, standards and directions as often as possible.

78 It is stressed that these expectations for proficiency will influence the assessment developed by individual teachers, with their subsequent influences on national assessments. Reporting to parents or to systems should be based on these expectations for proficiency. Indeed it is essential that the content strands, proficiency strands, pedagogy, assessment and reporting requirements are connected coherently.

CONCLUSION

79 While there is much commonality across jurisdictions and it will be possible to draw on the best of current curriculums, there are also some challenges for a national mathematics curriculum that are not resolved in current documents.

80 There will be substantial opportunities and challenges for teacher learning in the implementation of the national mathematics curriculum. Structuring a curriculum in the way that is proposed in this document will create a need for adjustments to some aspects of professional learning for mathematics teachers. In particular the emphasis will be on teachers understanding the big ideas of mathematics, as articulated in the curriculum, and then making active and interactive decisions on ways to teach that curriculum. This includes greater emphasis on finding out what the students know, and also greater emphasis on ways of adapting activities to enable access for students experiencing difficulty, and to extend students who may benefit from richer activities. Such emphases are compatible with current approaches to facilitate school- and classroom-based teacher learning.

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APPENDIX 1

The national curriculum: principles and specifications for development

The National Curriculum Board's work will be guided by the following principles and specifications for development:

a) The curriculum should make clear to teachers what has to be taught and to students what they should learn and what achievement standards are expected of them. This means that curriculum documents will be explicit about knowledge, understanding and skills and will provide a clear foundation for the development of a teaching program.

b) The curriculum should be based on the assumptions that all students can learn and that every child matters. It should set high standards and ensure that they apply to all young Australians while acknowledging the markedly different rates at which students develop.

c) The curriculum should connect with and build on the early years learning framework being developed for the pre-K phase.

d) The curriculum should build firm foundational skills and a basis for the development of expertise by those who move to specialised advanced studies in academic disciplines, professions and technical trades. It should anticipate and provide for an increase in the proportion of students who remain in education and training to complete Year 12 or equivalent vocational education and training and the proportion who continue to further study.

e) The curriculum should provide students with an understanding of the past that has shaped the society and culture in which they are growing and developing, and with knowledge, understandings and skills that will help them in their future lives.

f) The curriculum should be feasible, taking account of the time and resources available to teachers and students and the time it takes to learn complex concepts and ideas. In particular, the curriculum documents should take account of the fact that many primary teachers are responsible for several learning areas and should limit the volume of material which they must read in order to develop teaching programs.

g) The primary audience for national curriculum documents should be classroom teachers. Documents should be concise and expressed in plain language which, nevertheless, preserves a complexity in ideas appropriate for professional practitioners. Documents should be recognisably similar across learning areas in language, structure and length.

h) Time demands on students must leave room for learning areas that will not be part of the national curriculum.

i) The curriculum should allow jurisdictions, systems and schools to implement it in a way that values teachers' professional knowledge and reflects local contexts.

j) The curriculum should be established on a strong evidence base on learning, pedagogy and what works in professional practice and should encourage teachers to experiment systematically with and evaluate their practices.

(National Curriculum Board 2008:4)

APPENDIX 2: MATHEMATICS ADVISORY GROUP

The advice in this paper was provided by an advisory group led by Professor Peter Sullivan.

Professor Peter Sullivan, Professor of Science, Mathematics and Technology Education, Monash University

Dr Kim Beswick, Coordinator, Bachelor of Education Program, Education Faculty, University of Tasmania

Elizabeth Burns, Deputy Principal, Loreto Mandeville Hall, President of the Mathematical Association of Victoria

Dr Michael Evans, Project Manager, the International Centre of Excellence for Education in Mathematics

Sarah Ferguson, teacher, St Aloysius Catholic Primary School, Victoria

Dr David Leigh-Lancaster, Manager of Mathematics, Victorian Curriculum and Assessment Authority

Dr Thelma Perso, Executive Director, Curriculum Division, Department of Education, Training and the Arts, Queensland

Matt Skoss, teacher, Northern Territory

Dr Peter Stacey, Associate Dean (Academic), Faculty of Science, Technology and Engineering, Department of Mathematical and Statistical Sciences, Latrobe University

APPENDIX 3: FEEDBACK QUESTIONS

To provide us with feedback on this paper, please respond to the questions below. Your replies are a rich source of information and are of great value to us.

<table>
<tr><td colspan="4">Name:</td></tr>
<tr><td colspan="4">Organisation (if applicable):</td></tr>
<tr><td colspan="4">Postal address:</td></tr>
<tr><td colspan="4">Please nominate your area/areas of interest</td></tr>
<tr><td>☐ English</td><td>☐ mathematics</td><td>☐ science</td><td>☐ history</td></tr>
<tr><td colspan="4">Please choose:</td></tr>
<tr><td>☐ Academic</td><td>☐ Business or industry professional</td><td colspan="2" rowspan="4">Education professional
o Chief executive officer
o Curriculum director
o Curriculum manager
o Departmental/sector representative
o Principal
o Professional organisation representative
o School administrator
o Teacher
o Teacher’s aide</td></tr>
<tr><td>☐ Community member</td><td>☐ Journalist</td></tr>
<tr><td>☐ Parent</td><td>☐ Student</td></tr>
<tr><td>☐ Union representative</td><td>☐ Youth leader</td></tr>
</table>

If there is not enough space, please write on a separate sheet

Introduction

1. Please comment on the introduction.

Aims

2. To what extent do you agree with the aims of the mathematics curriculum?

☐ Strongly Disagree ☐ Disagree ☐ Agree ☐ Strongly Agree

3. Please comment

Terms used in this paper

4. To what extent do you agree with the definitions and applications of the terms used in the paper?

☐ Strongly Disagree ☐ Disagree ☐ Agree ☐ Strongly Agree

5. Please comment on the content and proficiency strands as organisers for the curriculum.

Considerations

6. Comment on the considerations that need to be taken into account when developing a national mathematics curriculum. Are there other considerations not canvassed in the paper?

Structure of the curriculum

7. The proposed structure identifies the curriculum focus for Stage 1 of schooling. To what extent do you agree with the focus in this stage?

☐ Strongly Disagree ☐ Disagree ☐ Agree ☐ Strongly Agree

8. Please comment

9. The proposed structure identifies the curriculum focus for Stage 2 of schooling. To what extent do you agree with the focus in this stage?

☐ Strongly Disagree ☐ Disagree ☐ Agree ☐ Strongly Agree

10. Please comment

11. The proposed structure identifies the curriculum focus for Stage 3 of schooling. To what extent do you agree with the focus in this stage?

☐ Strongly Disagree ☐ Disagree ☐ Agree ☐ Strongly Agree

12. Please comment

13. How many mathematics courses for the senior years of schooling should be included in the national mathematics curriculum?

14. Are there any other comments that you would like to make regarding the paper?

Send feedback

Post to: National Curriculum Board
PO Box 177
Carlton South VIC 3053

Fax to: (03) 8330 9401