

National Mathematics Curriculum: Initial advice

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National Curriculum Board's consultative process

Determining the form of the national curriculum

1 The National Curriculum Board is committed to an open development process with substantial consultation with the profession and the public. The Board began its consultation with the publication on its website (www.ncb.org.au) of *National Curriculum Development Paper*, a discussion paper in which it described the context of its work and set down a set of questions that it said it needed to answer to determine the kind of curriculum it would produce. That paper has been discussed at a national forum attended by 200 people on 27 June 2008 and in subsequent state and territory forums.

2 In the light of these discussions and its own further work the Board now sets down answers to its questions in the document *The Shape of the National Curriculum: a Proposal for Discussion*. That does not mean, however, that discussion is closed. The paper is posted on the Board's website with an invitation to anyone interested to provide comment and advice during Term 4 2008¹. After this time, the Board will determine its final recommendations and post them on its website in Term 1 2009.

Developing the scope and content of each national curriculum

3 The Board has also begun work on the shape of the national curriculum in English, mathematics, the sciences and history. For each, the Board recruited a writer who has worked with a small advisory group to draft a relatively brief initial advice paper that provides a rationale for students studying the curriculum and a broad scope and sequence of material to be covered over the years Kindergarten to year 12.

4 This approach will facilitate a discussion of the key issues in each curriculum before any detailed curriculum development commences. The first discussions will be held in the following national forums attended by 150-220 people:

| | |
|---------------------------|--------------------|
| Monday 13 October 2008 | <i>Science</i> |
| Tuesday 14 October 2008 | <i>Mathematics</i> |
| Wednesday 15 October 2008 | <i>History</i> |
| Friday 17 October 2008 | <i>English</i> |

5 At the forums there will also be some discussion about cross-curriculum learnings, including literacy and numeracy. Feedback from the forums will form part of the consultative process that will ultimately lead to more focused consultation about literacy and numeracy as a strong foundation for all learning, as outlined in the Board's remit to develop national curriculum.

6 On the day after each forum a small group of nominees from the relevant subject associations will meet with the authors and staff from the Office of the National Curriculum Board to provide their interpretation of the discussion in the forum and its implications for developing the curriculum. More detailed papers will be posted on the Board's website with an invitation to anyone interested to provide comment and advice in the period to 28 February 2009. After that, the Board will post on its website its final recommendations to guide curriculum development.

¹ Individuals can register on the website to receive email alerts when any new material is posted, particularly material on which comment and advice are invited.

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Professor Peter Sullivan biography

Professor Sullivan has been a teacher for 10 years, working in schools and universities in Papua New Guinea for six years. He has extensive experience in research and teaching in teacher education and has been a member of the Social, Behavioural and Economic Sciences panel of the Australian Research Council College of Experts for 2005 to 2007, and worked both at the Australian Catholic University and La Trobe University.

Currently he is Associate Editor, Journal of Mathematics Teacher Education, and Professor of Science, Mathematics and Technology Education, at Monash University.

National Mathematics Curriculum: Initial advice

Introduction

It is expected that the mathematics national curriculum will inform planning, teaching, and assessment of mathematics, and be useful and useable for both new and experienced teachers.

There have been two important reports this year relevant for this initial advice: the Australian *National Numeracy Review Report* (NNR, 2008); and *National Mathematics Advisory Panel* (NMAP, 2008) from the United States. Although both had some emphasis on what research says about learning, teaching, and teacher education, they both inform current considerations of the mathematics curriculum, particularly in describing the goals and intended emphases.

For the purposes of this discussion (only), the following terms are used:

- *Domain* describes the main and common content areas (e.g., number, space).
- *Topic* describes elements of each domain (e.g., place value, properties of shapes).
- *Strand* describes the aspects that go across domains (e.g., problem solving, critical thinking).

Goals of the mathematics curriculum and challenges facing schools

Building on the draft *National Declaration on Educational Goals for Young Australians*, a fundamental goal is that the curriculum should emphasize educating students to be informed thinking citizens, interpreting the world mathematically, appreciating the elegance and power of mathematical thinking, experiencing mathematics as an enjoyable experience, and using mathematics to inform predictions and decisions about personal and financial priorities. Further, as appropriate in a democratic society, many substantial community social and scientific issues are informed by public opinion, so there is also a need for broadly based capacity of citizens to interpret quantitative aspects of those issues.

A further goal is for Australia's future citizens to be sufficiently well educated mathematically to ensure international competitiveness. This has two aspects. The first is the need not only for adequate numbers of mathematics specialists operating at best international levels, capable of generating the next level of knowledge and invention, but also for mathematically expert professionals such as engineers, economists, scientists, social scientists, and planners. The second aspect is to produce an educated technical workforce contributing productively in an ever changing global economy, with rapid revolutions in technology and both global and local social challenges. Clearly, an economy competing globally requires substantial numbers of mathematically literate workers able to learn, adapt and create.

Yet there are challenges relevant to curriculum design that may inhibit the achievement of such goals. One is the widely reported student disengagement in the middle years of schooling which has been attributed to irrelevant curricula, ineffectual learning and teaching processes, and changing cultural and technological conditions (see Luke et al., 2003), which are particularly acute in the case of mathematics. A related challenge is that, while the overall proportion of students studying mathematics at Year 12 is steady, the decline in participation of students in specialised mathematics studies in most jurisdictions is a concern (Barrington, 2006). The NNR (2008) reported two linked issues: the first is a decline in students undertaking major sequences in tertiary mathematics study; and the second is the shortage of qualified secondary mathematics teachers, and

the resultant numbers of non specialist mathematics teachers in junior secondary years. Operating together these factors are cyclical: high quality teachers can support students in meaningful and productive mathematics learning in the key middle years, and so more students will retain aspirations for further mathematics study, and so it goes on.

The national mathematics curriculum needs to meet the needs of every student. There are two aspects to this. One is the need, given the obvious critical importance of mathematics achievement in providing access to further study and employment, to ensure that options for every student are preserved as long as possible. The second aspect of this challenge is the differential achievement among particular groups of students. For example, the following data are extracted from the report on the PISA 2006 results (Thompson & Bortoli, 2007) relating to numeracy (their actual term was *mathematical literacy*) of Australian 15 year olds, comparing the responses of the commonly discussed equity groups. Table 1 compares the achievement of students based on their socio-economic background:

Table 1: Percentage of Australian students from particular socio-economic backgrounds in highest and lowest levels of PISA numeracy achievement

| | Percent at the highest level | Percent at level 1 or below |
|-------------------|------------------------------|-----------------------------|
| Low SES quartile | 6 | 22 |
| High SES quartile | 29 | 5 |

Similar differences are evident when comparing non Indigenous and Indigenous achievement, and there are also differences in achievement levels between metropolitan, regional and remote students, and, to a lesser extent, between boys and girls.

The first point arising from the table is that SES background (and cultural and geographic factors in other data) seems very much related to numeracy (and other academic) achievement, which is contrary to a fundamental ethos of Australian education, that of creating opportunities for all students. While teachers have a critical role to recognise cultural, social or language factors contributing to such differences, the nature and emphases in the curriculum and subject choices are also important.

A second point is that it is extremely difficult to teach such a diverse range of students within the one class. Those not achieving PISA level 2 are responding at a very low level, and would have great difficulty coping with the demands of school as 15 year olds without specific support, and would have a restricted set of work choices available to them once they leave school. Yet those achieving at the highest level are progressing at the best international standards. The National Curriculum Board principles for the development of the curriculum noted that the top 10% of students are typically five years ahead of the bottom 10%. The spread is possibly greater for mathematics. It is tempting to address these issues by differentiating opportunities, but it is argued here that no barriers to progression in mathematics should be imposed, and students should have the opportunity to choose any mathematics study at the start of Year 10, and should not have their options restricted by their own previous choices or their school's structuring of subject offerings. It is the responsibility of systems and schools to provide the necessary additional support to enable this. The curriculum and supporting resources should also reflect this principle.

Embedding a futures-orientation in the curriculum

Incorporating a futures-orientation provides a major opportunity for the curriculum to lead improved teaching and learning. Specific decisions are needed on which aspects of the 20th century mathematics curriculum will be appropriate and which should be changed.

One of the considerations of this futures-orientation is that society will be complex, that workers will be competing in a global market, with a requirement that they know how to learn, adapt, create, and communicate, and with “intellectual curiosity, and the ability to find, select, structure and evaluate information” (Cisco Systems, 2008, p. 10). This perspective is also reflected in the draft *National Declaration on Educational Goals for Young Australians*, and the draft statement on mathematics for the 21st century being prepared by AAMT.

Another consideration is that digital technologies are not optional extras: they offer new approaches to explaining and presenting mathematics. The continuing evolution of digital technologies has progressively changed both school mathematics and the work of mathematicians (e.g., logarithm tables and the slide rule), and the curriculum must continue to adapt. Digital technologies are now more powerful, accessible, and pervasive. For example, modern mathematical technologies - hand-held devices or computer software - support numerical, statistical, graphical, symbolic, geometric and text functionalities. These may be used separately or in combination. Thus, a student could readily explore various aspects of the behaviour of a function or relation numerically, graphically, geometrically and algebraically using such technologies. There is potential for the curriculum and teaching programs to place less emphasis on rule based processing, and greater attention to meaning, transfer, connections and applications. Digital technologies can make previously inaccessible mathematics accessible, and enhances the potential for teachers to make mathematics interesting to more students, including the use of realistic data and examples. This is applicable at all levels of the curriculum, and is particularly relevant given the obvious capacity of school students to use digital, information and communication technologies.

It is recognised that some members of the community suspect that an increase in the use of technology will lead to a decline in levels of fluency, and lower standards of student achievement. A curriculum that appropriately incorporates mathematical technologies can anticipate such concerns by specifying standards for technology-free aspects of mathematics, and standards for those aspects where technologies enhance the quality of mathematics being learned. There is also opportunity for the process of developing the curriculum to educate the community about ways that technology supports learning of mathematics.

A subsidiary issue is that changes in usage of appropriate technologies happen slowly. There will be substantial implications for both prospective and practising teacher learning associated with the implementation of a curriculum in which mathematical technologies are embedded, and associated issues with the provision of the required hardware and resources.

Describing the strands of mathematics

Underpinning the curriculum will be a perspective on the nature of mathematics. When describing mathematics teaching, *The Cockcroft Report* (1982), in addition to listing teacher exposition, discussion and practical work as pedagogical approaches, recommended that mathematics teaching includes “consolidation and practice of fundamental skills and routines; problem solving, including the application of mathematics to everyday situations; and investigational work” (paragraph 243). These aspects help define the nature of the mathematics to be learned.

Similarly, *Adding it up* (Kilpatrick, Swafford, & Findell, 2001) described the following as five strands of mathematical content, with elaboration added:

- *conceptual understanding* includes the mathematical concepts and relations, the making of connections between related concepts, ways of thinking such as number sense, and the “why” as well as the “how” of mathematics

- *procedural fluency* includes skill in carrying out procedures flexibly, accurately, efficiently, and appropriately, and, in addition, factual knowledge and concepts that can be recalled readily
- *strategic competence* refers to what is commonly described as problem solving, including the ability to make choices, formulate and represent problem situations, and communicate solutions effectively
- *adaptive reasoning* includes the capacity for logical thought and activities such as proof, reflection, explanation, and justification, and what Christiansen and Walther (1986) described as “exploration, generalisation, description” (p. 262)
- *productive disposition* is an orientation to seeing mathematics as sensible, useful, interesting, elegant, and perhaps exciting, coupled with a belief in diligence and one’s self concept, which is argued by Marsh (2006) to be an important determinant of future learning.

It is argued that all of these must be part of the teaching and learning of mathematics. While the first two of these strands have commonly been the focus of many mathematics teachers and curriculum documents, the challenge will be to find the optimal way to incorporate the latter three strands. It will be especially challenging to decide which aspects should be explicitly presented in the curriculum as the content of mathematics. Aspects of these strands have been described as the essence of doing mathematics (see, for example, Crossley, 2006; Praeger, 2008). While some of these, especially *productive disposition*, are directly connected to pedagogies, the NCB explicitly asks that such process aspects be part of the documented curriculum.

Incorporating the numeracy perspective

An interesting observation on the *Adding it up* strands is that mathematical applications and numeracy are not prominent. A key issue for the curriculum will be to find ways to provide teachers and students guidance on standards for development of numeracy skills. Another key issue will be resolving the tension between recognising the importance of numeracy perspectives, and avoiding an artificial distinction between it and mathematics.

A definition of numeracy cited in the NNR (2008), based on the work of Willis (1992) and the *Ministerial Council for Education, Employment, Training and Youth Affairs* (MCEETYA, 1997) was:

Numeracy is the effective use of mathematics to meet the general demands of life at school and at home, in paid work, and for participation in community and civic life. (MCEETYA Benchmarking Task Force, 1997, p. 4)

Further, the *Australian Association of Mathematics Teachers* (AAMT, 1998) considered numeracy to be ‘a fundamental component of learning, discourse and critique across all areas of the curriculum’ (p. 1) and that it involves:

- the disposition to use, in context, a combination of: underpinning mathematical concepts and skills from across the discipline (numerical, spatial, graphical, statistical and algebraic); mathematical thinking and strategies; general thinking skills; [and] grounded appreciation of context. (p. 1)

The NNR (2008) endorsed these perspectives on numeracy, noting commonalities with approaches in the United Kingdom and The Netherlands, and identified numeracy as requiring an across-the-school commitment, including mathematical, strategic and contextual aspects. The curriculum will seek to develop numerate citizens, using these definitions.

In other words, numeracy is both an important aspect of the school curriculum, and sufficiently related to mathematics that their learning goals should be described together, even if distinctly. As the AAMT's (1997) description of the relationship between school mathematics and numeracy stated:

Numeracy is not a synonym for mathematics, but the two are clearly interrelated. All numeracy is underpinned by some mathematics; hence school mathematics has an important role in the development of young people's numeracy. The implemented mathematics curriculum (i.e., what happens in schools) has a responsibility for introducing and developing mathematics which is able to underpin numeracy. However this 'underpinning of numeracy' is not all that school mathematics is about. Learning mathematics in school is also about learning in the discipline – its structure, beauty and importance in our cultures. Further, while knowledge of mathematics is necessary for numeracy, having that knowledge is not in itself sufficient to ensure that learners become numerate. (pp. 11-12)

So it is argued that numeracy and mathematics be specified together. To do this, a more specific articulation of what is meant by the numeracy as part of the mathematics curriculum is needed. There appear to be two specific but different aspects of numeracy.

One aspect relates to capacities that enhance the lives of individuals by allowing them to see the world in quantitative terms, communicate mathematically, and interpret everyday information that is represented mathematically. It includes aspects such as number sense, basic measurement, such as length, mass, and capacity, estimating quantities, aspects of location including map reading, properties of shapes, personal finance and budgeting, and aspects of graphical interpretation.

A second aspect could be considered as vocational, and includes the mathematics used by professionals such as economists, architects, and engineers, the mathematics that is useful in learning disciplines such as psychology, geography, chemistry, physics, and electronics, and the everyday vocational mathematics used in fields such as building, sports, health, and catering. It involves aspects of accurate measurement, ratio, rates, percentages, using and manipulating formulas, the mathematics of finance, modelling and representing relationships especially graphically, and representing and interpreting sophisticated data.

Both aspects are clearly both about numeracy and about mathematics, and align with the draft *National Declaration on Educational Goals for Young Australians* outcomes related to building adult lives, and to "an ability to create new ideas and translate them into practical applications".

Associated with this suggestion that the development of numeracy and mathematics capacities be described together is a recognition of the contribution that numeracy perspectives make to the learning of other disciplines. For example, the following are indications of numeracy perspectives relevant to History, English, and Science:

History: Learning of history includes interpreting large numbers such as would be associated with population statistics and growth, financial data, figures for exports and imports, immigration statistics, war enlistments and casualty figures, both interpreting and representing a range of forms of data, imagining time lines and time frames to reconcile relativities of related events, and the visualisation required for historical imagination.

English: One aspect of the link with English and literacy is that, along with other objects of study, numeracy can be understood and acquired only within the context of the social, cultural, political, economic and historical practices to which it is integral. Another link between literacy and numeracy including the reading and writing demands of the formulation of practical problems, the challenge to decode information that can be interpreted mathematically, and capacity to communicate any mathematical products effectively. A further connection is through the meaning that can be made within texts by adequate interpretation of quantitative information similar to that mentioned for History above.

Science: Practical work and problem solving across all the sciences require the capacity to organize and represent data in a range of forms; plot, interpret and extrapolate graphs, estimate, solve ratio problems, use formulae flexibly in a range of situations, perform unit conversions, and use and interpret rates, scientific notation, and significant figures.

It is assumed that a comprehensive list of such capacities for the other disciplines be developed and included in the specification of those curricula.

In summary, this paper proposes that:

- learning in mathematics and numeracy are sufficiently related that they can best be described together, noting that curricula across Australia currently do not do this adequately
- other curriculum should specifically include, at relevant levels, those aspects of numeracy that are relevant for learning in those disciplines.

Purposes and emphases across stages of schooling

There should be an emphasis on the development of numeracy skills throughout the curriculum. Likewise, there should be a universal emphasis on appropriate strands of mathematical activity.

And it is essential that they are able to be implemented successfully and lead to an improvement on the status quo. For example, in a report of the Third International Mathematics and Science Study (TIMSS) Video Study, Hollingsworth, Lokan, and McCrae (2003) argued that in the Australian Year 8 lessons:

- more than three-quarters of the problems used by teachers were low in complexity (requiring four or fewer steps to solve)
- the majority of problems involved emphasis on procedural fluency
- only a quarter of problems used any real-life connections (42% in The Netherlands).

The National Curriculum should seek to change this. In this context, it is noted that the AAMT statements for both the early years and middle years emphasise the relevance of mathematics to the students' lives. AAMT and Early Childhood Australia (2006) emphasised that the young children can access 'powerful mathematical ideas' (p. 1) that are 'relevant to their current lives' (p. 1), and that it is the relevance to them of this learning that prepares them for the following years. Similarly, the AAMT (2005) vision for quality mathematics in the middle years reflects these emphases, noting the importance of students studying coherent, meaningful and purposeful mathematics that is relevant to their lives.

Consider, for example, the treatment of algebra. The NMAP (2008) directs significant attention on algebra (which is a separate subject in many places in the US), arguing that participation in algebra is connected to finishing high school, while failing to graduate high school is associated with under participation in the work force and high dependence on welfare. The NMAP argues that an emphasis in the primary years should be on preparation for algebra, and provides a detailed list of key topics in high school algebra (symbols and expressions, linear equations, quadratic equations, functions, algebra of polynomials, combinatorics and finite probability). On one hand, the study of algebra represents a challenge for many students during the compulsory years, and serves to exclude some students from further options. On the other hand, there are some students for whom the study of algebra is the first captivating aspect of mathematics, and the study of algebra clearly lays the foundations not only for specialised mathematics study but also for vocational aspects of numeracy. In other words, successful study of algebra is a gateway to opportunity, and unsuccessful study of algebra can be start of alienation from mathematics. A key aspect of the initial consultations will be to determine which aspects of algebra are best taught at which levels of schooling, what is the purpose of its study, and what should be its emphasis.

The senior secondary years

This section considers mathematics course arrangements at Year 12. (It is assumed that once there is agreement on the Year 12 offerings, Year 11 subjects could be designed subsequently.)

Despite the emphasis by some commentators on differences between the jurisdictions, there is substantial commonality in approaches to senior secondary mathematics study, as identified by Barrington (2006) who reported on enrolments in senior secondary mathematics courses and summarised the choices of students at three levels.

One level was for those students who study what Barrington termed an 'elementary' subject but not an intermediate or advanced subject. The largest enrolments of such students are in *General Mathematics* (NSW), *Further Mathematics* (Vic), and *Mathematics A* (Qld), each of which count toward tertiary selection.

Given that more than half of all Year 12 students choose these courses this seems an appropriate inclusion in a suite of national mathematics courses. Based on current arrangements the content might include topics such as business or financial mathematics, chance, data analysis and measurement, and in some places includes topics like navigation, matrices, networks, and applied geometry.

The next level of courses in the Barrington classification is the "intermediate', not advanced, level. The most common descriptor is *Mathematical Methods*, and other equivalent terms are *Mathematics* (NSW), *Mathematics B* (Qld), *Applicable Mathematics* (WA), and *Mathematics Studies* (NT). These courses are for students intending to study mathematics at university, and common topics include graphs and relationships, calculus, and statistics focusing on distributions. Some such subjects allow use of computer algebra system calculators as part of the teaching and learning.

The third level is described as 'advanced' mathematics, with the most common descriptor being *Specialist Mathematics*, and other terms being *Mathematics Extension* (1 and 2) (NSW), *Mathematics C* (Qld), and *Calculus* (WA). These courses are intended for those anticipating studying mathematics and engineering at university, and commonly include topics like complex numbers, vectors with related trigonometry and kinematics, mechanics, and build on the calculus from the intermediate subject.

It is noted that, while there are substantial variations across jurisdictions, there are many offerings at the senior secondary level designed for students pursuing vocational pathways but which are not used for university selection that can also be included in the recommendations at this level.

It is argued here that, rather than seeking some consensus or compromise between the offerings across the jurisdictions, the debate should define the purposes of senior secondary mathematics study for the various course levels, after which the courses can be designed to address those purposes. Nevertheless the three Barrington levels, along with a non tertiary entrance vocational option, provide a useful starting point for discussion.

Finding clear and succinct ways to describe the curriculum

The form of presentation of the curriculum will be critical to its successful implementation. The experience of many users of curriculum documents in the various jurisdictions is that they are long, complex, written in convoluted language, with ambiguous category descriptors, in which it is difficult to identify key ideas. The *Australian Primary Principals Association* (APPA, 2008) noted the high workload demands that current curriculum specifications place on teachers.

Documents will be written so that they are accessible to teachers, and also assist in defining a language for communication about the curriculum nationally; noting that the current diversity in terminology adds to complexity and means that many of the high quality resources produced in various jurisdictions are difficult to transfer to other contexts.

It is also important that the curriculum communicates powerfully and succinctly the important ideas of the curriculum, and the key phases through which these progress. There are compelling reasons for this. Hattie and Timperley (2007) reviewed a wide range of studies and found that teacher feedback to students is a key determinant of effective learning. Hattie and Timperley explained that feedback involves making explicit to students what they should be doing, how they are performing, and what is the next phase in their learning. Teachers do this while they are interacting with students, and so need to know the purpose of the current student activity, the expected standards for performance, and subsequent learning goals. A thoughtfully, powerfully, and succinctly written curriculum will assist in this.

Consider, for example, a possible description of the key steps in learning subtraction. (Note that it is not claimed that this is the only or optimal way of teaching subtraction in these years, but it is offered as an attempt to clarify what is intended by “succinct description”.) Assuming that children have experienced preliminary ideas including counting forwards, ordering numbers, seeing that there is the same number of objects however they are arranged (conservation), and immediate recognising of small collections of objects (subitising), the key phases of learning subtraction could be described as:

- counting backwards
- one part of the whole is hidden
- useful number strategies that help subtraction
- solving subtraction word problems
- efficient subtraction with larger numbers.

It is argued that this is a substantially simplified way of describing the steps in learning subtraction than is used in most curricula, but these steps are comprehensive as well as being succinct and give teachers clear intentions of the purpose of each step.

The levels at which the curriculum is represented is another key organisation issue. Presenting the curriculum in stages or phases, such as K-2, 3-4, 5-6, 7-8, 9-10 and 11-12, could be seen to encourage teachers to recognise that they are dealing with students whose levels of development are spread over more than one year. In contrast, the APPA (2008) argued that the curriculum be presented by year of schooling. It is also noted that the response in some places to specifying the curriculum in levels has been for subsequent delineation of further details of the progression of student learning.

Certainly there seems to be advantages in specifying the expected standards of student achievement for the levels of the national assessments of student performance noting that this is every two years.

Streamlining the curriculum

The NMAP (2008) argued, 'the mathematics curriculum in Grades PreK-8 should be streamlined and should emphasise a well-defined set of the most critical topics in the early grades' (p. 11). When all aspects are presented as though of equal importance, this does not help teachers to appreciate short and long term goals, and to identify the key ideas. It is possible to reduce some of the crowding by addressing complementary concepts together, but there may still be a need for the identification of core topics, or other mechanisms that can allow teachers to feel less hurried.

It may be possible to use the sequential specification of the curriculum to reduce some of this sense of being rushed. The NMAP (2008) noted that the mathematics curriculum is marked by 'effective, logical progressions from earlier, less sophisticated topics into later, more sophisticated ones' (p. xvii). The panel also argued that 'a focused, coherent progression of mathematics learning, with an emphasis on proficiency with key topics should become the norm of elementary and middle school mathematics curricula. Any approach that continually revisits topics year after year should be avoided' (p. xvi). While it is recognised that not all students progress through aspects of mathematics in the hypothetical optimal sequence (Denvir & Brown, 1986), that the pathway from one key idea to another is not the same for all students, and that students tend not to have consistent performance across all aspects of mathematics (Gervasoni, 2004), the optimal way to specify all key learning goals is to do this sequentially. It is recognised that this also is a complex task, and is not a feature of most current curricula.

Connected to the notion of sequence is the issue of the pace of development. For the purposes of curriculum documentation, learning for most topics can be considered to be a continuum, although not necessarily with development at a regular rate. The tension is whether to set an ultimate goal for secondary school mathematics (such as calculus) and develop the learning continuum to meet the goal, or to establish an optimal sequence and pacing and have the goal determined by how far the students develop.

Setting high but realistic expectations for student achievement

The curriculum will establish expectations for student achievement. Some guidance can be obtained from the statements for the curricula in high achieving countries, particularly Singapore and Finland, although these propose topics for study at lower year levels than those in Australia. Recognising that a frequent response by students is that the mathematics they study is not challenging, that a common characteristic of schools that are successful is the high expectations they set, and that there are many reports of students of good teachers who exceed current curriculum expectations, there is a need to consider the level of expectations to be set by the curriculum.

Yet there may be disadvantages in specifying curriculum targets at unrealistically high levels in comparison with expected student achievement. If, for example, teachers are seeking to teach topics at levels for which the students are not ready, this can result in teachers emphasising rote methods and students missing the opportunity to build the connections that would make their learning robust. There would be substantial advantages for student learning if they studied content thoroughly, and achieved what the NMAP (2008) described as closure. Meaningful and challenging problems can be posed using mathematical tools with which the students are familiar, and so having realistic expectations does not necessarily result in students having less challenging learning experiences. The curriculum may not need to develop quickly or be set at an advanced level for the best students to be challenged. The goal is not to have an advanced curriculum, but well educated students.

It is noted that there are now substantial data from which realistic expectations can be determined. It would be possible, for example, to analyse student responses to relevant items on large scale assessments such as NAPLAN, and from large scale projects (e.g., Clarke et al., 2002), and to use that information to ensure that the curriculum describes realistic expectations for student learning. Such analysis may also assist in determining topics which are not currently well taught (e.g. fractions) and adapting the way such topics are presented in the curriculum.

Categorising and describing the content

The intention is to describe a curriculum in ways that will support teachers in their work. One approach is to consider the content as multidimensional, and specifically describe at least two of these dimensions: the basic domains and the strands.

Domains

Although there are some variations and combinations, the most common categorisations of the basic domains of the curriculum across the jurisdictions in the compulsory years are:

- Number
- Measurement
- Space
- Chance and data
- Algebra

The last category is sometimes used only at the secondary level, while some jurisdictions use terms like *Structure* or *Pattern* to describe those elements of algebraic thinking relevant in the primary years.

Strands

The other dimension would be the strands that apply across these domains. Nearly all jurisdictions use the category 'working mathematically' as though it is an additional domain. There are two reasons that this is inadequate: first, working mathematically actually applies to the other domains rather than being its own domain; and second, the working mathematically category does not adequately acknowledge other aspects which also apply across the domains. The form of presentation should seek to resolve this issue.

The elements that could be considered within this other dimension include:

- Conceptual understanding
- Procedural fluency
- Strategic thinking
- Adaptive reasoning
- Productive disposition

- Numeracy
- Language
- Technology

Finding a way to represent these strands adequately will assist the curriculum writers.

Conclusion

This process of consultation, which commences at the forum on 14 October, is an important opportunity for our community to engage with these issues in the interests of future Australian learners of mathematics.

Looking towards implementation

There is potential in the use of technology for delivery of the curriculum in ways that can enhance the presentation of the descriptors, and any subsequent elaborations. This is especially important since the curriculum specification will need both to indicate the sequential progression of the key ideas, as well as the connections between ideas especially at the one level.

There will also be some teachers who prefer succinct description, and others who may prefer additional elaboration. There will be those who hope that the curriculum be presented by year of schooling, by combinations of years as is common across jurisdictions, or in some other grouping. All options may be possible using information and communication technology (ICT). Of course, paper versions would also be made available.

The effective use of ICT as the medium of publication may assist in developing a flexible, adaptable, and scalable curriculum, with systems, schools and teachers being able to choose their own form and format of presentation. A flexible form of publication will allow linking to relevant supporting resources, and encourage the development of those resources.

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